AN Estimation of Frontier Production Functions and the Technical Efficiencies Of Wheat Farmers of the Gezira Scheme

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Abstract

A frontier production function of Cobb-Douglas type is used to estimate the technical efficiency of the wheat tenants of the Gezira Scheme.

The technical efficiency for the farm operator is defined in terms of the ratio of its mean production to the corresponding mean production if the farm effect was zero.

The model is applied in the analysis of farm level data from a random sample of the wheat tenant farmers in the Gezira Scheme.

Estimates of the productive efficiency of individual tenant farmers are found to be very law, ranged between 0.274 and 0.713, and the bulky of them – about 52% scored a rate of technical efficiency less than or equal to 0.400.

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1-INTRODUCTION

Frontier production functions have been the subject of considerable econometric research during the last decades. Theoretical and applied modeling of production process has resulted from the seminal work of Farrell (1957). Farrell suggested that a production function should be estimated as envelop function associated with combination of observed inputs. Aigner and Chu (1968), Timmer (1971), Afriat (1972) and Schmidt (1976) made important contribution to the econometric modeling of frontier production functions. However, the model proposed independently by Aigner, Novell, and Schmidt (1977), and Meeusen and van den Broeck (1977) represented and important breakthrough. Given the assumptions of this model it was possible to obtain estimates for the precision of estimators for the parameters of the frontier production functions and perform various tests of hypotheses on those parameters.

An important component of frontier production functions is the subject of the technical efficiency of individual farms. In early studies of frontier production functions involving cross-sectional data on sample farms, the technical efficiency of a given farm was defined as the ratio of the observe production to the corresponding frontier value associated with the given farm's factor inputs. Jondrow et al (1982) obtained important results by which best predicators of the technical efficiency of individual firms could be obtained. Battese and Coelli (1988) considered a generalized frontier production function for time series data on sample farm operators and defined predicators for the technical efficiencies of individual farmers which are a generalization of the results obtained by Jondrow et al (1982).

In this paper we consider the Battese-Coelli model for which different numbers of observations are available on the sample tenants. The model is applied to data obtain from a random sample drawn from wheat farmers of the Gazira Scheme.

2- THE GENERALIZED FRONTIER MODEL

We consider the frontier production function

$$Y_{it} = \exp(X_{it}\beta + V_{it} - U_i) \qquad \dots\dots\dots$$

i

$t=1,2,\dots\dots\dots,T_i$, $i=1,2,\dots\dots\dots\dots N$;

where:

Y_{it} is the output for the t^{th} observation on the i^{th} sample tenant;

X_{it} is (1xk) vector of functions of values of inputs associated with the t[th] observation on the i[th] sample producer;

is a (Kx1) vector of unknown parameters,

T_i represents the number of observation on the i[th] tenant.

N represents the number of the sample tenants for which observation are available; and the V_{it} random variables are assumed to be independent and identically distributed as $N(O,\sigma_v^2)$ random variables, independent of the non-negative U_i random variables, which are assumed to be independent and identically distributed truncations (at zero) of the distribution.

The frontier production function (3-10) is of Cobb-Douglas (C-D) type and the first element of the vector X_{it} is assumed to be one, and the remaining elements are (associated with) values of inputs of different variables included in the model. The tenant effects in the model, ,

i=1,2,….., N, are assumed to be time-invariant random variables. It is assumed that they are non-negative random variables which are associated with the presence of technical inefficiency of the sample farmers. The statistical properties of the tenant effects are assumed to be defined by truncating (at zero) a general normal distribution which mean is unknown.

The assumption that the tenant effects are time-invariant may be unsatisfactory for many empirical situations, particularly when the period of time involved are quite large, in addition the distributions of the tenant farmer effect may depend on the levels of inputs of the tenant. These problems are not consider in this study.

3- FARM - LEVEL TECHNICAL EFFICIENCY [2]

Following Battese and Coelli (1988) the technical efficiency of the i^{th} sample tenant farmer is defined as the ratio of the mean production for the i^{th} farmer, given its realized tenant effects, and its level of inputs, to the corresponding mean production if the tenant effect was zero. Thus if the technical efficiency of the i^{th} tenant is denoted by T , then

$$TE_I = \frac{E(Y_{it}|U_i, X_{it}.X_{it}, i=1,2,....)}{E(Y_{it}|U_i=0, X_{it}.i=1,2,...)}$$

[2] The following representation follows that of Battese, Coelli and Colby (1989)

Given the frontier production function (I), it follows that the technical efficiency of the i[th] tenant farmer reduces to:

………..III

The predication of the technical efficiency (III) requires distributional results for the tenant effect U_i conditional on the sample observations on production. If the parameters of the production frontier model are assumed known, then it is possible to derive the conditional distribution of the tenant effect, U_i , conditional on the random vector, E_i, where:

$$E_{it} \equiv (E_{i1}, E_{i2}, \ldots \ldots . E_{iT_i})$$

and $E_{it} \equiv V_{it} - U_i, \qquad i = 1,2, \ldots \ldots \ldots, T_i.$

It can be shown, given the assumptions of the frontier production function (I), that the conditional distribution of U_i, conditional on E_i , is defined by the truncation (at Zero) of the

, where:

$$\mu_i^* = \frac{\mu\sigma_v^2 - \sigma^2 T_i \bar{E}_i}{\sigma_v^2 + T_i\sigma^2} \qquad \ldots\ldots\ldots\ldots \text{ IV}$$

$$\sigma_i^{2*} = \frac{\sigma^2\sigma_v^2}{\sigma_v^2 + T_i\sigma^2} \qquad \ldots\ldots\ldots\ldots\ldots \text{ V}$$

and $$\bar{E}_i \equiv \frac{1}{T_i}\sum_{E=1}^{T_i} E_{it}$$

It is evident that, as the number of observations, on the i^th tenant farmer increases, the value of the mean, μ_i^* approaches U_i and the variance, σ_i^{*2}, approaches zero.

It is well known that the best (minimum squared error) predicator of an unobservable random variable, conditional on the value of a known random variable, is the conditional expectation of the first random variable, conditional on the value of the second random variable. Thus the best predicator of the technical efficiency of the i^th tenant farmer, exp ($-U_i$), conditional on the value of random vector, E_i, is E(exp($-U_i$)/E_i). Given the above distributional assumption, it can be shown that:

$$E[\exp(-U_i)\,|E_i] = \left\{\frac{1-\Phi[\sigma_i-(\mu_i/\sigma_i)]}{1-\Phi(\mu_i/\sigma_i)}\right\} exp\left(-\mu_i+\frac{1}{2}\sigma_i^2\right) \ldots\ldots VI$$

where $\Phi(.)$ represents the distribution function for the standard normal random variable

When the parameters of the frontier production function are unknown, the suggested predicator for the technical efficiency of the i^th tenant farmer is:

$$\widehat{TE}_i = \frac{1 - \Phi[\hat{\sigma}_i^* - (\hat{\mu}_i^*/\hat{\sigma}_i^*)]}{1 - \Phi(\hat{\mu}_i^*/\hat{\sigma}_i^*)} exp\left(-\mu_i + \frac{1}{2}\hat{\sigma}_i^*\right) \ldots\ldots\ldots VII$$

where:

……..VIII

……IX

$$\hat{\bar{E}}_i = \frac{1}{T_i}\sum_{i=1}^{T_i}\hat{E}_{it}, \hat{E}_{it} = \log(Y_{it}) - X_{it}\hat{\beta} \cdots\cdots\cdots X$$

and $\hat{\beta}, \hat{\mu}, \hat{\sigma}^2$ And $\hat{\sigma}_v^2$ represent the maximum –likelihood estimators (MLE) for the parameters of the frontier production function (I)

Given the frontier production function (I) the mean technical efficiency of tenant farmers in the sector is given by:

$$TE = \frac{1 - \Phi[\sigma - \left(\frac{\mu}{\sigma}\right)]}{1 - \Phi\left(\frac{\mu}{\sigma}\right)} exp\left(\mu + \frac{1}{2}\sigma^2\right) \ldots\ldots XI$$

This can be estimated by substituting the maximum –likelihood estimators for the parameter, μ and σ^2 , of the distribution of the tenant effects for the unknown parameters in ( ).

4-THE DATA SETS

This study involved drawing a random sample from wheat farmers of the Gazira Scheme in the agriculture season of 2002-2003 using three-stages random sampling, where the 18 administrative groups of the scheme were accepted as being a natural geographical clustering of primary sampling units (PSUs). Six groups were selected according to probability proportional to size. Each of the selected 6 groups is divided into 6-11 blocks, i.e. the second-stage units (SSUs) were actually 44 blocks. The second-stage of this multi-stages sampling was a random selection of a block out of each of six groups, i.e. the second-stage units were six blocks. Selected blocks or SSUs consisted of a number of tenants from whom the final sample was selected. A predetermined sample of 200 wheat farmers was divided between the six blocks according to the weight of total population in each block. Then the weighted sub-sample was randomly selected from each block, and the final sample was arrived at by the six sub-sample. However , only 130 tenants farmers were interview[3]

[3] - The limitation of the final sample to 130 tenants was governed by the available number of enumerators, hours of work, transportation, and financial problems.

The data used in the analysis were collected from all wheat growers in the sample (130 tenants). These data included information about labour, fertilizer, tenancy size, output, yield, capital, and prices. The statistics on the variables used in the estimation are reported in table (1).

The variable capital is the weighted average of the cost of machinery operations, insecticides, and irrigation, the weights being the share of each component in the total. The variable labour includes both family and hired labour.

Table (1): Summary Table :

Variable measure	Mean	Standard Deviation	Minimum	Maximum
Labour (man-hour)	7.6	3.62	2.8	10.4
Fertilizer (Kilograms of nutrient)	972.86	750.31	130.75	5404.00
Capital (S.Dinar)	19575	9546.11	15600	24806
Land (Feddans)	5.00	3.85	2.25	15.00
Output (ton /fedd.)	0.720	0.149	0.194	1.300
Yield (Kilogram/fedd.)	0.720	0.149	0.194	1.300
Revenues (S.Dinar/ton)	65130	486.73	58140	70806
Price of Fertilizer(S.D /fedd)	7580	3801.50	6870	7925
Price of labour (S.D/fedd.	12888	8652.14	10724	15021

Source: Field Survey, 2003

5- EMPIRICAL APPLICATION

We consider the frontier production function

$$+$$

$$\beta_2 \mathrm{Ln}B_{it} + \beta_3 \mathrm{Ln}F_{it} + \beta_4 \mathrm{L} \quad \text{xi}$$

Where Y_{it} is the total wheat output for the i^{th} tenant farmer in the t^{th} year of observation; D_{it} is the wheat cultivated area for the i^{th} farmer in the t^{th} year of observation; $B_{it} \equiv a_1$ FB_{it}, + (1- a_1) HB_{it} is a labour variable in which FB_{it} and HB_{it} are the hours of family and hired labour (in male equivalent units) for the i^{th} tenant farmer in the t^{th} year of observation, where a is a parameter between 0 and 1; S_{it} is the seed rate applied by the i^{th} farmer in the t^{th} year of observation; K_{it} is the value of capital, for the i^{th} tenant farmers in the t^{th} year of observation; F_{it} is quantity of fertilizer applied by the i^{it} tenant farmer in the t^{th} year of observation; and the random variables V_{it} and U_{it} have the distributional properties as stated for the model (1).

The parameter of human labour model in the frontier production function was estimated by the value of a_1 which minimize the residual sum of squares for the production function (xi) the value of $\hat{a}_1$= 0.613 (with estimated standard error of 0.044) has the smallest residuals for the model (xi). The co-efficient of determination (adjusted) for this model was 0.869, which indicate that the model accounts for a high proportion of the variability in the logarithm of wheat yield for tenant farmers in the Gezira Scheme.

The above estimate for the human labour model in the frontier production function (xi) implies that the estimated labour model is equivalently expressed by:

$HB_{it} = FB_{it} + 0.63HB_{it}$ ………… xii

where HB is the human labour, and the estimate 0.63 is (1- $\hat{a}_1$)/ $\hat{a}_1$.

The estimated aggregate human labour model (xii) is such that one hour of hired male labour is approximately equivalent to 0.63 of an hour of family male labour. However, the estimated weighting factor, 0.63, in the labour model (xii) is significantly different from one. Alternatively, the estimate $\hat{a}_1$= 0.613, is significantly different from 0.05 at five- percent level.

Since the parameter, a_1, of the labour variable in the frontier production function (xi) was significantly different from 0.05 (at five- percent level), we conclude that hired and family labour are not homogeneous in the Gezira Scheme.

The frontier production function (xi) was estimated. The estimated frontier model (xi) was:

$$LnY_{it} = 4.66 + 0.51LnD_{it} + 0.61LnB_{it} + 0.65LnF_{it} + 0.18Lnl$$
xiii

(0.88) (0.27) (0.26) (0.094) (0.38)

The estimated value of the parameter, μ , which defines the distribution of the farm effects for the different tenant farmers was 0.12830 with an estimated standard error of 1.637.

The distribution of the farm effects was found to be significantly different from the half normal distribution in which μ = 0, significant at the one-percent level.

$$\hat{\gamma} = 0.46718, \qquad \hat{\bar{\sigma}} = 0.28171$$

(0.581) (0.063)

where the parameters γ and $\hat{\sigma}^2$ are defined by

$$\gamma = \sigma^2/(\sigma^2 + \sigma_v^2)$$

$$\hat{\sigma}^2 = (\sigma^2 + \sigma_v^2)$$

The estimate for the parameter γ ,which is the ratio of the variance σ^2 , involved in defining the distribution of the farm effects U_i, to the sum of the two variances σ^2 and σ_v^2, is smaller than the estimated standard error. However, the generalized likelihood ratio test for the hypothesis γ is zero had value 6.58, which was significant at the one percent level. This indicates that the frontier production function is significantly different from the traditional Cobb-Douglas production function which does not involve non-negative farm effects.

Given the frontier production function (xiii) model for the total wheat yield for tenant farmers sample, it follows that the use of the predictor (vii) obtains estimates for the technical efficiency of the sample tenant farmers. The predicted efficiencies are given in table (2). The predicted efficiencies range from 0.274 to 0.718. The frequency of values of efficiencies in different ranges is indicated in table (4-8). 52.3% of the sample of tenants farmers had predicted efficiencies less than 0.400 where as only 6.6% of the farmers had predicted efficiencies greater than or equal to 0.700. The estimate for the mean efficiency (equation 3-20) of wheat tenant farmers in the Gezira Scheme is 0.586.

Table (2): Frequency Distribution of Predicted Efficiencies for the Sample Tenants in the Gezira Scheme.

Range of Efficiencies	**Frequency of Occurrence**	**Relative Frequency of Occurrence (%)**
Less than 0.300	32	29.9
0.300 and less than 0.400	24	22.4
0.400 and less than 0.500	18	16.8
0.500 and less than 0.600	14	13.1
0.600 and less than 0.700	12	11.2
Greater than or equal to 0.700	7	6.6
Total	107	100.0

Source: Field Survey, 2003.

6- CONCLUSION

A basic component of frontier production functions studies is the prediction of technical efficiencies of individual farms operators, and identification of factors which contribute significantly to high levels of technical efficiencies.

The empirical application if frontier production functions to farm-level data from the Gezira Scheme indicates that the productive efficiency of wheat tenant farmers in the scheme is very low.

Our empirical application also show that family and hired labour are not homogeneous, given the model considered for labour services family labour should be weighted has 1.5 times as much as hired labour.

It worth mentioning that the frontier production function I considered in this analysis has the property of the proportionality of the variance of production and the square of the mean of production. Moreover, as to Just and Pope (1979) the model has the property that increasing the level of variable results in an increase in the variance of production if elasticity is positive. Alternative types of frontier production functions are to be examined in future empirical studies.

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